Session 10
Summary and Review

1.0 Introduction

The previous sessions showed that Newton's Laws of Motion are used during aircraft flight testing. During this final session, the same techniques used to evaluate a full size aircraft are used to predict the performance of a radio controlled (R/C) model aircraft. The scope of testing is limited because there is no pilot on board and instrumentation, such as airspeed and fuel flow, is not available. As a result, the focus of this session is on weight and balance, thrust determination, and takeoff performance. The procedures described can be accomplished by any student having access to a R/C model.

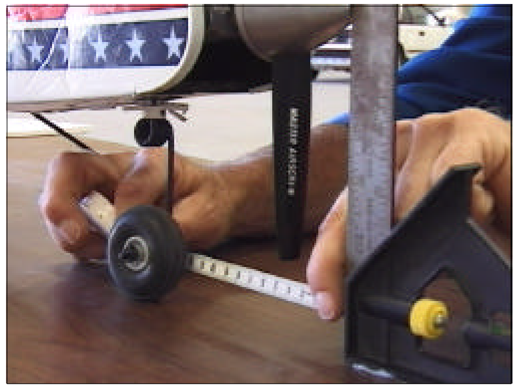
2.0 Weight and Balance of the Model

To determine the aircraft's weight and location of the center of gravity, use the same procedures described in Session 2. Begin by establishing a Reference Datum Line (RDL) at the forward end of the propeller hub. This can be done by placing a carpenter's square at the end or by placing the model flush against a wall. From the RDL, measure the horizontal distance to the point where the nose wheel (or tail wheel, depending on the type of model) touches the ground. This is the arm length for the nose gear. Accomplish the same procedure for the main landing gear.

NOTE:
Since the assumption is that the aircraft is symmetric, only one main gear need be measured.

The following lengths were found using the model shown in the video:

Nose Landing Gear Arm = 5.75 inches
Main Landing Gear Arm = 14.25 inches



Measuring Landing Gear Arms

The aircraft is then weighed. Recall from Session 2 that a scale is placed under each landing gear, the weights are recorded and then added together to obtain the total aircraft weight.

Caution:

It is important that the aircraft be level to achieve the proper weight distribution on each landing gear.

Weighing the Aircraft

Now, to determine the *cg* location, the weight recorded at each landing gear is multiplied by the arm length from the RDL for that gear. For the model under evaluation, this yields the following:

Item (Gear)	Arm	Weight	Moment
Nose	5.75 in	0.88 lbs	5.06 in-lbs
Left Main	14.25 in	1.75 lbs	24.93 in-lbs
Right Main	14.25 in	1.56 lbs	22.23 in-lbs
Total		4.19 lbs	52.22 in-lbs

Then, divide the total moment by the total weight to determine the location of the *cg*. Here, the *cg* is located 12.46 inches from the RDL, which locates it on the wing within the range of *cg*'s specified by the model maker.

3.0 Determining Thrust of the Model

The next step in the flight test sequence is to predict the thrust available from the engine. Session 5 gave the relation:

$$T = \left[\rho\left(\frac{\pi d^2}{4}\right)\right]2\left[2Vk(RPM) + k(RPM)^2\right]$$

The model maker has provided the following information for use in this equation:

Propeller diameter (d): 9 inches (0.75 ft)
Propeller efficiency (k): 0.00066
Max engine RPM: 1250 RPM

NOTE:
For most models, propeller constants range between 0.00044 and 0.00070. Tests to determine actual k values are very involved. Therefore, should you decide to conduct a test similar to the one shown in the video, an average propeller constant of 0.00057 can be used.

The air density at the test site is 0.002 slugs/ft^3. By examining the equation, we see a velocity term in the second set of brackets. However, for a static thrust check, the velocity is zero. Therefore, substituting into the thrust relationship gives:

$$T = \left[0.002\frac{slugs}{ft^3}\left(\frac{\pi 0.75ft}{4}\right)^2\right]2\left[0 + 0.00066(1250)^2\right]$$

$$T = 1.82 \text{ lbs}$$

To verify this value, a spring scale is attached to the model. With the engine operating at maximum RPM, the scale reading is 1.5 lbs. To predict the takeoff performance, the 1.5 pounds of static thrust determined experimentally should be used in the calculations.

Measuring Engine Thrust

NOTE:
This is 21% less power than predicted. This highlights an important aspect of testing. The numbers for RPM and propeller efficiency provided by the manufacturer are for a brand new engine under carefully controlled test conditions. The engine on the model is a number of years old and the propeller has a considerable number of "nicks" on the blades. Each of these factors detracts from the amount of thrust the engine can produce. This is why we test the thrust using a scale.

4.0 Determining Takeoff Speed

Recall from Session 6 that the lowest speed at which the lift just equals the aircraft's weight is the takeoff speed. This speed is determined by the relationship:

$$w = L = \tfrac{1}{2}\rho V^2 S C_{L_{max}}$$

Rearranging this equation to solve for takeoff speed gives:

$$V = \left(\frac{2w}{\rho S C_L}\right)^{\frac{1}{2}}$$

In order to determine the wing area, the video depicted measuring;

- the chord length, *c*, (distance from leading edge to the trailing edge of the wing)
- the wing span, *b*, (distance from one wingtip to the other)

To determine the area of the rectangular wing, simply multiply the chord length times the span length, or

$$S = c \times b$$

For the model being tested, the chord length is 8.5 inches (0.7083 ft) and the wing span is 52 inches (4.33 ft). Multiply these values and we find the wing area is 3.07 square feet.

Determining Wing Area

Caution:

It's important to convert all units into feet and pounds prior to performing the calculations for takeoff speed, lift, drag, and thrust.

The next item needed for the takeoff speed calculation is the air density. For the temperature and air pressure measured on the day of the test, the density, ρ, was found to be 0.002 slugs/ft^3. This may vary for your test. All we need now is the overall lift coefficient.

The lift coefficient is usually found by wind tunnel analysis. In this case, the model maker didn't provide this data. A conservative number for an aircraft without flaps and a rectangular wing is C_L = 1.1. This is a reasonable assumption and can be used for most model applications.

Applying these numbers to the takeoff speed equation:

$$V = \left(\frac{2w}{\rho S C_L} \right)^{\frac{1}{2}}$$

$$V = \left(\frac{2(4.19 lbs)}{\left(0.002 \frac{slugs}{ft^3}\right)(3.07 ft^2)(1.1)} \right)^{\frac{1}{2}}$$

$$V = 35.2 \text{ ft/sec (24 MPH)}$$

This speed will be used in determining the drag on the aircraft during the takeoff roll.

5.0 Determining the Drag

Session 6 said that engineers have learned through experience, if seventy percent of the takeoff speed is used to calculate the drag during takeoff, the results are very close to the average drag. 70% of the speed just calculated is 0.70 times 35.2 ft/sec or 24.6 ft/sec.

Next the drag coefficient, C_D, should be determined. Again, this is usually found in a wind tunnel. However, for the type of model used in this test, a good estimated value of C_D is 0.06.

The drag equation is:

$$D = \tfrac{1}{2} \rho V^2 S C_D$$

Using the values for wing area, density, 70% of takeoff speed, and drag coefficient, the predicted average drag during the takeoff is:

$$D = \tfrac{1}{2}\left(0.002 \frac{slugs}{ft^3}\right)\left(24.6 \frac{ft}{\sec}\right)^2 (3.07 ft^2)(0.06)$$

$$D = 0.1115 \text{ lbs}$$

This value and the measured value for thrust are used to calculate the expected acceleration during takeoff.

6.0 Determining the Acceleration

Using Newton's $F = ma$ equation, we can define acceleration in the same manner as outlined in Session 6. This yields:

$$F = T - D = ma$$

when we rearrange the equation, we can solve for the acceleration:

$$a = \frac{T - D}{m}$$

$$a = \frac{g(T - D)}{w}$$

Inserting the appropriate values (remember $g = 32.2 \frac{ft}{\sec^2}$) gives:

$$a = \left(32.2 \tfrac{ft}{\sec^2}\right)\left(\frac{1.5lbs - 0.1115lbs}{4.19lbs}\right)$$

$$a = 10.67 \tfrac{ft}{\sec^2}$$

To determine the time required to accelerate to takeoff speed, use the following relationship;

$$time = \frac{takeoff\ speed}{acceleration\ rate}$$

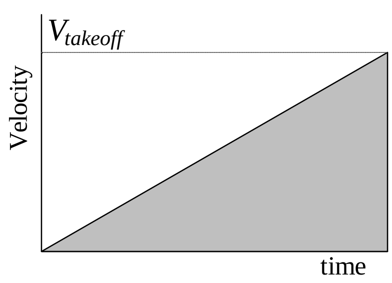
$$time = \frac{35.2 \frac{ft}{\sec}}{10.67 \frac{ft}{\sec^2}}$$

$$time = 3.3 \sec$$

In Session 6, it was stated if we assume the acceleration rate is constant, a plot of velocity versus time can be constructed. The acceleration rate is simply the slope of this curve. So if we use this relationship, the takeoff distance is determined.

7.0 Determining Takeoff Distance

Since the acceleration is assumed to be constant, the slope of the plot is a straight line. Using the right triangle equation, Session 6 showed that the area under the triangle is equal to the estimated takeoff distance.



$$area = \tfrac{1}{2}(base \times height)$$

$$area = \tfrac{1}{2}(takeoff\ speed \times time\ required\ to\ takeoff)$$
$$= takeoff\ distance$$

Substituting the appropriate values into this equation:

$$area = \tfrac{1}{2}(35.2 \tfrac{ft}{\sec} \times 3.3 \sec)$$

$$area = takeoff\ distance = 58\ ft$$

During the first takeoff of the model, the takeoff distance was measured at 85 feet. To account for the increased takeoff roll, we must account for rolling friction.

Rolling Friction

From Session 6, the rolling friction is given as:

$$Friction = \mu w$$

where μ is the coefficient of friction for the surface the aircraft is rolling over. The surface of the "runway" used in the video is dirt with rocks and holes throughout. The handbook value of μ is 0.1, for surface conditions of the runway. To account for friction we use Newton's equation again:

$$F = (T - D - \mu w) = ma$$

and calculate the new acceleration rate:

$$a = \frac{g(T - D - \mu w)}{w}$$

$$a = \frac{32.2 \frac{ft}{\sec^2}[1.5lbs - 0.115lbs - 0.1(4.19lbs)]}{4.19lbs}$$

$$a = 7.45 \tfrac{ft}{\sec^2}$$

So, to estimate the new time required to accelerate to takeoff speed:

$$time = \frac{takeoff\ speed}{acceleration\ rate}$$

$$time = \frac{35.2\frac{ft}{\sec}}{7.45\frac{ft}{\sec^2}}$$

$$time = 4.72 \text{ sec}$$

Taking these factors into consideration, the new estimated takeoff distance should be:

$$area = \tfrac{1}{2}(takeoff\ speed \times time\ required\ to\ takeoff)$$
$$= takeoff\ distance$$

$$area = \tfrac{1}{2}(35.2\tfrac{ft}{\sec} \times 4.72 \sec)$$

$$takeoff\ distance = 83.1 \text{ ft}$$

This takeoff distance was within 2 feet of the actual distance required for the first takeoff. On a subsequent takeoff the distance required was 86 feet. This further substantiates our analysis.

8.0 Conclusion

The techniques used to flight test aircraft rely heavily upon Newton's Three Laws of Motion. Although some simplifying assumptions have been made to the aerodynamic relationships, the basic concepts remain valid regardless of the size of the aircraft. We demonstrated this by testing of a R/C Model. Further experiments are outlined in the section titled "Culminating Activities." We hope you find them interesting and challenging.